Properties of Fifth Powers

a last digit of a a mod 9	a^5	last digit of a^5	a^5 mod 9
9	59049	9	x
8	32768	8	8
7	16807	7	4
6	7776	6	0
5	3125	5	2
4	1024	4	7
3	243	3	0
2	32	2	5
1	1	1	1
0	0	0	0

Example 1
For a three-digit integer a, we are given $\boldsymbol{a^5}$ **= 362,59082,03125**. Can we find the three-digit $a = d_1d_2d_3$ in our heads (without use of paper or calculator)?

Step 1. Since the last digit of a^5 is 5, the last digit of a must be $d_5 = 5$, or a = $d_1d_2 5$.

Step 2. If a were 300, then a^5 = 243,00000,00000. If a were 400, then a^5 = 1024,00000,00000. Since a^5 = 362,59082,03125 lies between these, the first digit d_1 must be 3, or $a = 3d_2 5$.

Step 3. Since a^5 mod 9 = 1, then it must be that a mod 9 = 1. What d_2 would cause $a = 3d_2 5$ to have a mod 9 of 1. We see 325 mod 9 = 1, so
***a* = 325** is correct.

a last digit of a a mod 9	a^5	last digit of a^3	a^3 mod 9
9	59049	9	x
8	32768	8	8
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6	7776	6	0
5	3125	5	2
4	1024	4	7
3	243	3	0
2	32	2	5
1	1	1	1
0	0	0	0

Example 2

For a three-digit integer a, we are given $\boldsymbol{a^5}$ **= 35518,34554,15293**. Find the three-digit $a = d_1d_2d_3$.

Step 1. Since the last digit of a^5 is 3, the last digit of a must be $d_5 = 3$, or $a = d_1d_23$.

Step 2. If a were 800, then a^5 = 32768,00000,00000. If a were 900, then a^5 = 59049,00000,00000. Since a^5 = 35518,34554,15293 lies between these, the first digit d_1 must be 8, or $a = 8d_23$.

Step 3. Since a^5 mod 9 = 0, then it must be that a mod 9 = 0, 3, or 6. What values of d_2 would cause $a = 8d_23$ to have a mod 9 of 0, 3, and 6?
For d_2 = **7**, then 873 mod 9 = 0. For d_2 = **1**, then 813 mod 9 = 3. For d_2 = **4**, then 843 mod 9 = 6. To decide between d_2 = 1, 4, and 7, we need to go back to Step 1. Was 35518 close to 32768, close to 59049, or in the middle? Since 35518 is close to 32768, we chose d_2 = 1, and the answer is $\boldsymbol{a}$ **= 813**.