

Answers 5

Problem 1

$$1942/7 = 277.428571\dots \quad 0.428571 \times 7 = 3$$
$$1942/13 = 149.384615\dots \quad 0.384615 \times 13 = 5$$

Problem 2

The smallest number divisible by both 7 and 13 is $7 \times 13 = 91$

Problem 3

Numbers with a mod 7 of 1	Numbers with a mod 13 of 6
1	6
8	19
15	32
22	45
29	58
36	71
43	
50	
57	
64	
71	

So $71 \bmod 7 = 1$, and $71 \bmod 13 = 6$.

Similarly, it's also true that $162 \bmod 7 = 1$, and $162 \bmod 13 = 6$, but I'm probably not 162.

Problem 4

$$1836542 \bmod 9 = 2. \quad 315734 \bmod 9 = 5.$$

Problem 5

$$1942 \bmod 10 = 2. \quad 1942 \bmod 100 = 42.$$

Problem 6

$$1942 = a \times 7^3 + b \times 7^2 + c \times 7^1 + d \times 7^0 = a \times 343 + b \times 91 + c \times 7 + d \times 1, \text{ where } a, b, c, \text{ and } d \text{ are all 6 or less.}$$

$$1942/343 = 5.661807\dots \text{ so } a = \mathbf{5}. \quad 1942 - 5 \times 343 = 227.$$

$$227/91 = 2.494505\dots \text{ so } b = \mathbf{2}. \quad 227 - 2 \times 91 = 45.$$

$$45/7 = 6.428571\dots \text{ so } c = \mathbf{6}. \quad 45 - 6 \times 7 = 3.$$

$$3/1 = 3, \text{ so } d = \mathbf{3}.$$

Then $1942_{10} = 5263_7$. The last "digit" is 3, which agrees with $1942 \bmod 7 = 3$ in Problem 1.

Problem 7

$$X = a \times n + x, \text{ and let } Y = b \times n + y$$

$$X \bmod n = x, \text{ and } Y \bmod n = y$$

$$X \times Y = (a \times n + x)(b \times n + y) = a \times b \times n \times n + a \times y \times n + b \times x \times n + x \times y = (a \times b \times n + a \times y + b \times x) \times n + x \times y$$

$$[X \times Y] \bmod n = [(a \times b \times n + a \times y + b \times x) \times n + x \times y] \bmod n = [x \times y] \bmod n = [X \bmod n \times Y \bmod n] \bmod n$$

Problem 8

1942	1942 mod 9 = 7	7	
<u>×385</u>	385 mod 9 = 7	<u>×7</u>	
747670	747670 mod 9 = 4	49	49 mod 9 = 4