Answers 3

Problem 1

If you ask whether it has spots, that divides the bugs in half (8). Then if you ask whether it has a blue body, that divides the remaining bugs in half again (4). Then if you ask whether it has a blue head, that divides the remaining bugs in half again (2). Then if you ask whether it has six legs, that divides the remaining bugs in half again (1), and you know the answer. For example, if your friend thinks of Ralph, then yes it has spots, no it doesn't have a blue body, yes it has a blue head, and no it doesn't have six legs.

The problem was designed so it doesn't matter what order you ask the four questions. Try it. That wouldn't be the case if all the bugs with a blue body also had six legs, for example.

Problem 2

With 20 Questions, you should try to always divide the remaining possibilities in half. If it's animal, ask first "Is it a vertebrate?" This doesn't divide the animal kingdom in half since there are so many types of bugs. But people are less familiar with all the kinds of bugs, and they're very familiar with vertebrates. So it's probably a good question.

There are $2^{20} = 1,048,576$ different patterns of yes/no answers. There are about the same number of patterns that are shorter than 20 answers. So the best you can do is come up with about 2 million different answers. So whether you can win or not depends on how many things your friend knows and on how good you are at dividing each remainder in half.

Problem 3

p_5	p_4	p_3	i_{10}	p_2	i_9	i_8	i_7	p_1	i_6	i_5	i_4	i_3	i_2	i_1	i_0	
?	?	?	0	?	0	1	1	?	0	1	0	0	1	1	1	
								1	0	1	0	0	1	1	1	(p_1 chosen for five 1s)
				0	0	1	1					0	1	1	1	(p_2 chosen for five 1s)
		0	0			1	1			1	0			1	1	(p_3 chosen for five 1s)
	0		0		0		1		0		0		1		1	(p_4 chosen for three 1s)
0	0	0	0	0	0	1	1	1	0	1	0	0	1	1	1	(p_5 chosen for seven 1s)

Problem 4

p_5	p_4	p_3	i_{10}	p_2	i_9	i_8	i_7	p_1	i_6	i_5	i_4	i_3	i_2	i_1	i_0	
1	1	0	1	0	1	1	0	0	1	1	1	0	1	1	0	
								0	1	1	1	0	1	1	0	(five 1s. Error **not** in the right half.)
				0	1	1	0					0	1	1	0	(four 1s. Error is in the right half.)
		0	1			1	0			1	1			1	0	(five 1s. Error **not** in the right half.)
	0		1		1		0		0		0		1		0	(three 1s. Error **not** in the right half.)
0	0	0	1	0	1	1	0	0	1	1	1	0	1	1	0	(the errored bit is p_2; it should be a 1)

So the i bits were actually correct:

$$11101110110_2 = 2^{10} + 2^9 + 2^8 + 2^6 + 2^5 + 2^4 + 2^2 + 2^1$$

$$= 1024 + 512 + 256 + 64 + 32 + 16 + 4 + 2 = 1910$$

Problem 5

Every bit other than p_5 is in the field of at least one of the other p bits. So if it is errored, at least one of the parity fields will have even parity, and you will know there's an error without looking at p_5. If p_5 is errored, none of the other p bits will know about it, but who cares, it's not an information bit.

So we could throw away the p_5 bit, making the block length smaller without affecting the number of information bits. This would improve the efficiency a small amount. But it's so nice to deal with a block that's a power of 2 that nobody worries about the small gain in efficiency.

Problem 6

A block of length $2^n = 2$ needs $n + 1 = 2$ parity bits, so the number of information bits is zero, and the efficiency is zero. For 1 information bit, we need $n = 2$ and a block length of $2^n = 4$. But as shown in Problem 5, the highest parity bit is unnecessary, and the block length can be reduced to 3. Then 111 represents a 1, and 000 represents a 0.