Answers 6

Problem 1
a^5 = **9,84658,04768** $a = d_1d_2d_3$

Step 1. Since the last digit of a^5 is 8, the last digit of a must be $d_5 = 5$, or a = $d_1d_2$8.

Step 2. If a were 100, then a^5 = 1,00000,00000. If a were 200, then a^5 = 32,00000,00000. Since a^5 = 9,84658,04768 lies between these, the first digit d_1 must be 1, or a = 1$d_2$8.

Step 3. Since a^5 mod 9 = 2, then it must be that a mod 9 = 5. What d_3 would cause a = 1$d_2$8 to have a mod 9 of 5. We see 158 mod 9 = 1, so a = **158**.

Problem 2
a^5 = **1886,65362,36032** $a = d_1d_2d_3$

Step 1. Since the last digit of a^5 is 2 the last digit of a must be $d_5 = 5$, or a = $d_1d_2$2.

Step 2. If a were 400, then a^5 = 1024,00000,00000. If a were 500, then a^5 = 3125,00000,00000. Since a^5 = 1886,65362,36032 lies between these, the first digit d_1 must be 4, or a = 4$d_2$2.

Step 3. Since a^5 mod 9 = 5, then it must be that a mod 9 = 2. What d_3 would cause a = 4$d_2$2 to have a mod 9 of 2. We see 452 mod 9 = 2, so a = **452.**

Problem 3
a^5 = **92721,65023,65625** $a = d_1d_2d_3$

Step 1. Since the last digit of a^5 is 5, the last digit of a must be $d_5 = 5$, or a = $d_1d_2$5.

Step 2. If a were 900, then a^5 = 59049,00000,00000. If a were 1,000, then a^5 = 1,00000,00000,00000. Since a^5 = 92721,65023,65625 lies between these, the first digit d_1 must be 9, or a = 9$d_2$5.

Step 3. Since a^5 mod 9 = 7, then it must be that a mod 9 = 4. What d_3 would cause a = 9$d_2$5 to have a mod 9 of 4. We see 985 mod 9 = 4, so a = **985.**

Problem 4
a^5 = **11964,33987,33024** $a = d_1d_2d_3$

Step 1. Since the last digit of a^5 is 4, the last digit of a must be $d_5 = 4$, or a = $d_1d_2$4.

Step 2. If a were 600, then a^5 = 7776,00000,00000. If a were 700, then a^5 = 16807,00000,00000. Since a^5 = 11964,33987,33024 lies between these, the first digit d_1 must be 6, or a = 6$d_2$4.

Step 3. Since a^5 mod 9 = 0, then it must be that a mod 9 = 0, 3, or 6. What values of d_3 would cause a = 6$d_2$4 to have a mod 9 of 0, 3, and 6?
For d_2 = **8**, then 684 mod 9 = 0. For d_2 = **2**, then 624 mod 9 = 3. For d_2 = **5**, then 654 mod 9 = 6. To decide between d_3 = 8, 2, and 5, we need to go back to Step 1. Was 11964 close to 7776, close to 16807, or in the middle? Since 11964 is about half-way between, we chose d_2 = 5, and the answer is a = **654**.

Problem 6

a last digit of a a mod 9	a^3	last digit of a^3	a^3 mod 9
9	729	9	x
8	512	2	8
7	343	3	1
6	216	6	0
5	125	5	8
4	64	4	1
3	27	7	0
2	8	8	8
1	1	1	1
0	0	0	0

$\boldsymbol{a^3}$ **= 146,363,183**. $a = d_1d_2d_3$

Step 1. Since the last digit of a^3 is 3, the last digit of $a = d_1d_2d_3$ must be $d_3 = 7$, or a = $d_1d_2$7.

Step 2. If a were 500, then a^3 = 125,000,000. If a were 600, then a^3 = 216,000,000. Since a^3 = 146,363,183 lies between these, the first digit d_1 must be 5, or $a = 5d_27$.

Step 3. 146,363,183 mod 9 = 8, so a^3 mod 9 = 2, 5, or 8. What values of d_3 would cause $a = 5d_27$ to have a mod 9 of 2, 5, and 8? For d_2 = **8**, then 587 mod 9 = 2.
For d_2 = **2**, then 527 mod 9 = 5. For d_2 = **5**, then 557 mod 9 = 8. To decide between d_3 = 2, 5, and 8, we need to go back to Step 1. Was 146 close to 125, close to 216, or in the middle? Since 146 is close to 125, we chose d_2 = 2, and the answer is ***a* = 527**.