Can Discovery and Intuition Be Taught?

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Abstract – **A course on teaching engineering provided an opportunity to investigate how to teach through discovery. One of the results was to organize the method into four steps: the teacher discovers, realizes how he or she discovers, guides the student in a similar experience, and uncovers why the student did so poorly. Following this method, the students in the class showed a natural skill in using discovery when dealing with math puzzles, but they lost that sense when dealing with their engineering discipline. We look at the reasons for this and list roadblocks that keep discovery from being generally taught. This paper looks at methods to shorten the time and increase the success of teaching discovery. It seeks a balance in which some material is presented in the traditional lecture mode, but some material the students are allowed to discover for themselves.**

Index Terms – creativity, design, discovery, insight, intuition.

INTRODUCTION

I don't know the answer to the question posed by the title of this paper; there is evidence on both sides. The purpose of the paper is to help the reader answer the question for himself from his or her own perspective.

On the one hand there seems to be a clear consensus that teaching discovery and intuition is of the highest importance. To me, the heart and soul of learning and engineering is discovery, and I hear the same from my colleagues around the lunch table. Respected educators, such as Jerome Bruner, support the importance of discovery in learning:

> Mastering the fundamental ideas of a field involves not only the grasping of general principles, but also the development of an attitude toward learning and inquiry, toward guessing and hunches, toward the possibility of solving problems on one's own. … To instill such attitudes by teaching requires something more than the mere presentation of fundamental ideas. Just what it takes to bring off such teaching is something on which a great deal of research is needed, but it would seem that an important ingredient is a sense of excitement about discovery—discovery of regularities of previously unrecognized relations and similarities between ideas, with a resulting sense of self-confidence in one's abilities. Various people who have worked on curricula in science and mathematics have urged that it is possible to present the fundamental structure of a discipline in such a way as to preserve some of the exciting sequences that lead a student to discover for himself.[1]

Jack Lochhead observes, "Teachers who have the well-developed habit of explaining everything as clearly as they can, find it hard to hold back and let the students learn/discover for themselves. Yet it is essential that they do this, if the students are to take a more active role in their own learning."[2] Richard Feynman says discovery is the motivation and reward: "The prize is the pleasure of finding the thing out, the kick in the discovery…"[3]

On the other hand, the importance of discovery seems more honored in the breach than in the observance. Few learning experiences in an engineering curriculum (except, perhaps, the capstone design experience) include much discovery on the student's part. Classes and labs deal almost exclusively with nomenclature, equations, properties of components, and analysis tools. I myself allow far fewer opportunities for the student to discover than I give lip service to. So it seems that, despite our high ideals, some realities keep discovery from being incorporated in practice. If that's true, then teaching discovery and intuition in the class room is practically impossible.

Last year I taught a course on teaching engineering, and I decided to tackle the issue head-on. I made teaching through discovery the main focus of the class. My successes and failures in that class are the basis for this paper.

DISCOVERY AND INTUITION

For our purposes here, discovery is finding the solution to a problem on the basis of incomplete knowledge. As an example, suppose a communications engineer is to transmit patterns using the bits 1 and 0. She knows that with n bits she can generate 2^n different patterns. But the physical limitations of the transmitter don't allow the transmission of two adjacent 1s. How many patterns are possible with n bits now? For n = 1, there are the patterns 0 and 1; for n = 2, there are 00, 01, and 10; for n = 3 there are 000, 100, 010, 001, and 101; for n = 4 there are eight patterns. She lays out a table of the results so far, where $p(n)$ is the number of patterns:

n	1	2	3	4
$p(n)$	2	3	5	8

The sequence for $p(n)$ reminds her of the Fibonacci series, so she guesses that $p(5) = 13$. Counting the bit patterns for $n = 5$ confirms this. Now, the Fibonacci series is formed by

$$p(n) = p(n-1) + p(n-2), \quad p(1) = 2, \; p(2) = 3. \qquad (1)$$

It's impossible to prove that this holds for all n by trying all ns, but the engineer finds a proof of the iterative equation (1) from studying the bit patterns.

The engineer's discovery was based on incomplete knowledge in that she didn't have an algorithm that led her directly to the solution. But it did rely on familiarity with (knowledge of) the Fibonacci series, iterative equations, and inductive proof. With less background knowledge, the solu-

tion would have taken longer and depended more on luck. At the other extreme is nearly complete knowledge. For example, one could set the problem: Given equation (1), discover the value of $p(n)$ for $n = 15$. Now almost all the work has been done—almost everything is known from the outset. It would be a stretch to call this a discovery.

Because knowledge is incomplete, the discoverer necessarily needs to employ *intuition* and *heuristics*. Both terms refer to the use of nonrigorous methods to solve problems, but "intuition" usually means that the methods and background knowledge are used unconsciously. Bruner lists several heuristic rules—the use of analogy, the appeal to symmetry, the examination of limiting conditions, the visualization of the solution.[4] Billy Vaughn Koen goes so far as to say "everything the engineer does in his role as an engineer is under the control of a heuristic."[5] This implies that if an engineer is just plugging numbers into equations to achieve a design, he is not really acting as an engineer.

Discovery is not limited to classes on design—the ultimate problem solving that engineering students are being trained for. Discovery can (and should regularly) be employed in classes dealing with fundamentals such as control theory or Bernoulli's principle. Can the student be led to (re)discover feedback as Harold Black did?[6]

THE FOUR STEPS IN TEACHING DISCOVERY

From my own experience with discovery and with trying to lead students to discovery, I had assembled a list of important tools—guessing, visualization, specific-to-general, simplify, look at examples with real numbers, use leading questions, listen carefully to the student. I wanted to incorporate these into the course on teaching engineering, but I was looking for a structure that would organize these tools. About a week into the course I realized that they were all part of four steps necessary to teaching discovery:

1. Develop the habit of playing around with ideas and problems to discover insights and solutions.
2. Be aware of the steps taken in the discovery—what background knowledge, analogies, and similar problems were used.
3. Devise ways to lead the student to a similar discovery through questions and exercises, leaving as much as possible to the student.
4. Assess what the student lacked—why he needed so much help. Be good at listening and getting inside the student's mind.

The process is iterative, going back to Step 3, or possibly to Step 1, until the student needs little help.

In Step 1 the teacher gets good at what he is to teach. The term "playing around" includes the recognition of patterns and similarities to other ideas and problems. Therefore the degree of success is dependent in large part on accumulated experience. (It's estimated that an expert has about 50,000 "chunks" of knowledge to bring to bear on a new problem.[7]) Many books are available that deal with heuristics of discovery.[1,3,8,10,12] The natural ability to see patterns and the love of discovery seem to be almost universal. But the emphasis here is on being habitually curious and trying things out. The repeating decimal for 1/7 repeats every six digits; is it possible for a fraction $1/n$ to repeat at a period of more than $n - 1$ digits? Why is the repetition length always $n - 1$ (or some factor thereof) when n is prime?

Step 2 is what separates teachers from engineers. An engineer gets a solution, implements it, and moves on to the next problem. A teachers needs to reflect on how he solved the problem—to be self-aware. So (with apologies to G.B. Shaw[9]) we can say that those who can, do, and those that know how they can, teach. Being aware of how we solve problems is not easy. The process is fuzzy and moves rapidly, with tenuous connections and alternative attempts. It's difficult to be aware of the concepts we're using, as the term "intuition" implies. So step 2 takes practice to do it well. It helps to think out loud and have someone write down your thoughts[10] (or write them down yourself later). George Polya has provided examples of the thinking behind discovering solutions to some mathematical problems[11].

Step 3 has long been recognized as a tool in teaching. Socrates' name[12] has been given to the method of teaching by using questions to lead the student. In recent times, George Polya has perhaps best advocated the use of questions in prodding students to discover solutions for themselves (see his book *How to Solve It*[13]). Another way to promote discovery is to devise experiences for the students—homework, labs, and projects. Almost every course in engineering includes these elements, but the challenge is to design and order them so as to lead to a discovery experience—one in which the students uncover a surprise and gain insight. The exercises should neither give too much away nor leave the student stymied. It might be necessary to set the student up with an experience that serves as a clue for a later problem.

Step 4 is necessary because Step 3 is almost always a failure to some degree. How could the students have failed to recognize the Fibonacci series? In looking for patterns in a sequence, isn't taking the difference between elements one of the first things you try? What is lacking in the students' experience? Were they taking a different (and perhaps promising) tack than you expected? Some of the evaluation in Step 4 can take place in grading the homework and exams (if the teacher does the grading rather than teaching assistants). But the mistakes usually don't reveal the underlying misconceptions. To get to the root of the students' problems, it is essential to have discussions in which the teacher does more listening than talking. Because students don't want to expose their ignorance, the teacher must put them at ease and provide an atmosphere for honest exchange. When the students can't really say what's troubling them, the teacher must probe with questions and use his imagination to guess the problem.

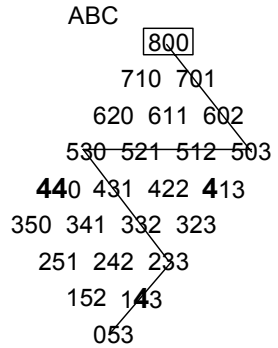
The four steps need constant revising. With iteration, things improve, but always short of satisfactory results because time with the students is short. As with most things in life, the illusion is that we should be able to finally get it perfect and then settle down and relax. This is, fortunately, never true.

THE SUCCESSES

The students in the class on teaching engineering were from various disciplines—electrical, mechanical, chemical, and civil. Therefore we began with problems in mathematics—a discipline common to all. Each student chose a math puzzle and presented it to the class, applying all four steps in leading the class to discover a solution. (I suggested puzzles rather than standard problems, such as finding the volume of a sphere, which have pat answers.) I could observe the presenters' performance in carrying out Steps 3 and 4, and I could infer whether they had properly prepared in Steps 1 and 2.

The results were satisfying; the presenters had good puzzles and came prepared with heuristics that would help towards a solution. The questions used in Step 3 and the probing in Step 4 (when the questions failed to lead) could have been better, but they were commensurate with a first-time effort. The main success was that the students showed that they had done well with Steps 1 and 2—with playing around and monitoring their own thought process. For example, one student presented the problem associated with equation (1) above. (He posed it: "If a teacher arranges students in a row of seats so that students don't sit next to each other, how many arrangements are there for *n* seats?")

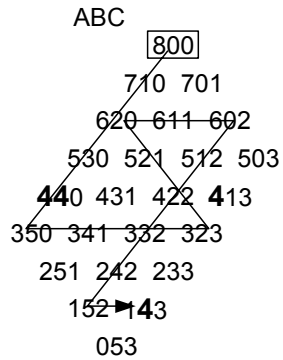
Another student presented the following puzzle. You have an 8-oz. container filled with water, an empty 5-oz. container, and an empty 3-oz. container. The object is to pour the water back and forth until you have 4 oz. in a container. There are no graduations on the containers, so you have to pour until the receiving container is full or the delivering container is empty. The class played around with pouring sequences, but it was clear that some way to keep track of the moves was necessary. The presenter suggested that we could list all the possible states of water distribution by 1-oz. increments. The class came up with the following graph:



The 8-, 5-, and 3-oz. containers are labeled A, B, and C, respectively. At the top of the graph, all 8 oz. of water are in container A. In the second row 1 oz. has been poured into container B (left) or container C (right), reducing container A to 7 oz. Starting with state 710 in the second row, states in the third row are generated by pouring an additional 1 oz. from A to B (the state 620) or from A to C (the state 611). Proceeding in this manner, all 24 possible states are generated. Notice that rows have constant values for A, one diagonal has constant values for B, and the other diagonal has constant values for C.

By playing around, the class found that "legal" moves amounted to moving along a row or a diagonal until they hit a boundary. The graph above shows four moves as straight lines. Starting at state 800, the first move pours 3 oz. from A to C, filling C. The second move pours 3 oz. from C to B, emptying C. The third move pours 3 oz. from A to C, filling C again. The fourth move pours 2 oz. from A to B, filling B and emptying A.

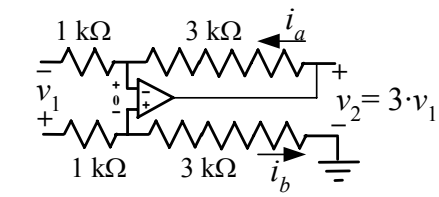
The target states are 440 or 413 or 143. It didn't take the class long to determine that the fewest number of moves from 800 to a target state is six, as shown below. The graph provided an excellent way to keep track of the moves and to trace possible paths quickly with the eye. (What is the fewest number of moves to get from 800 to 413?) By the time they were done, the class felt that they had found all possible solutions. That would not have been possible with just blind trial-and-error (with no knowledge or structure).

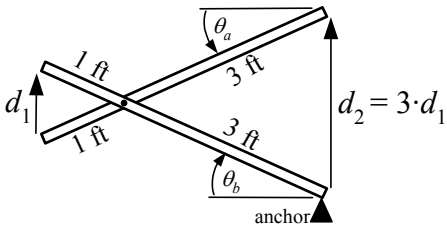


THE FAILURES

After some practice at teaching discovery through the use of math puzzles, we moved on to engineering disciplines. Each student was to take a topic from his or her discipline and present it in a manner of discovery to the class. The expectation was that most class members would not be too familiar with the presenter's topic. In any case, the presenter was to take a nontraditional approach—one that would surprise even someone in that discipline.

The presenter was to avoid specialized nomenclature and use specific numerical examples to lead to general rules (equations). The problem should be simplified at first and then increased in complexity to introduce new concepts one at a time. These shouldn't seem to be "pulled out of a hat," but follow naturally from fundamental concepts— $f = ma$, conservation of energy, charge, and mass. The results should be made to feel familiar (intuitive) by the use of examples, analogies, and geometry—electric current is like water flow; the number of 20-bit sequences is like the number of objects identifiable in the game "twenty questions;" the output phasor of a first-order low-pass filter traces out a semicircle as the frequency is swept. The diagram below shows an analogy between a differential amplifier circuit and a pair of levers configured like scissors; voltage is analogous to height and current to angle.





The "discovery" problems the students chose to present were disappointing. A graduate student in civil engineering presented the following as a discovery problem:

> *Flow* (q) = hourly rate of vehicles passing a point (veh/hr).
> *Mean velocity* ($\bar{u}_s$) = harmonic mean of the velocities of vehicles.
> *Density* (k) = number of vehicles in a unit length of highway at an instant (veh/mi).
> Relationship: $q = k\bar{u}_s$.
>
> A change of flow from q_1 to q_2 causes a backup wave moving at a velocity
>
> $$u_w = \frac{q_2 - q_1}{k_2 - k_1}, \qquad (2)$$
>
> where k_1 and k_2 are the initial and final densities.
>
> **Problem**: 1600 veh/hr approach a signal-controlled intersection at 25 mph and a density of 76 veh/mi. When the signal turns red, cars come to a stop with a density of 116 veh/mi. Discover the wave velocity of the queue backup.
>
> **Solution**:
>
> $$u_w = \frac{q_2 - q_1}{k_2 - k_1} = \frac{0 - 1600 \text{ veh/hr}}{116 - 76 \text{ veh/mi}} = -40 \text{ mi/hr}$$

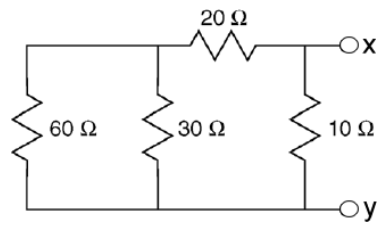
The equation (2) for u_w was pulled out of a hat! Whoever developed the equation has already exercised most of the discovery. The work remaining is just plug-and-grind, which is hardly a discovery. Evidently, this is the way the concept was taught to the presenter, and she had a hard time seeing it any other way.

The definition of *harmonic mean* and how it is applied is another whole issue. Is the mean taken over a mile or over an hour? The problem doesn't use mean velocity, so this concept could have been put off until later. It's better to simplify and treat one concept at a time.

A better problem would be to discover equation (2) by looking at a number of examples. Suppose all cars are 20 feet long and are traveling 60 ft/sec in a single lane at a spacing of 80 feet (front bumper to front bumper). If they come to a stop instantly upon touching the rear bumper of the car ahead, what is the wave velocity of the backup? The class should be able to figure this out without equation (2). Then look at a problem where, instead of stopping, the speed is halved to 30 ft/sec and the density is doubled to a spacing of 40 feet (front bumper to front bumper). What is the new wave velocity? An analogy to water flowing in a rubber pipe might lend insight. In the end, the class will have developed equation (2) and have some insight as to why it is true. I wrote down my thought process in following this line of discovery and put it on the Web.[14]

A graduate electrical engineering student presented the following as a discovery problem:

> Resistors R_1 and R_2 in series can be replaced by a resistor of value $R_1 + R_2$.
> Resistors R_1 and R_2 in parallel can be replaced by a resistor of value $(R_1 \cdot R_2)/(R_1 + R_2)$.
>
> **Problem:**
> Discover the resistance between terminal x and y in the circuit below.



> **Solution:** 60||30 = 20, 20 + 20 = 40, 40||10 = 8 Ω.

This problem is a little better in that there are still some decisions left—where to start, what to combine next. But, again, most of the interesting discovery has been done in developing the equations for series and parallel. I asked the presenter where the equations came from—why they held. She said it was obvious that resistors in series add, and the parallel equation came from the reciprocal of the sum of reciprocals. I asked why the equations were obvious, and she said she didn't know; she had used them so long they just felt right.

A better discovery problem would be to look at parallel 60-Ω and 30-Ω resistors and find the equivalent resistance by using the definition $R \equiv V/I$. (This could be motivated by an analogy to water flow though a narrow pipe developing a pressure difference.) Using conductance rather than resistance may lend some insight. After another parallel example with different values, the student could generalize for R_1 and R_2 and come up with the parallel equation. Then develop the series equation in a similar manner. Finally the student would be ready to tackle the problem in the diagram above. In this manner, the problems build on each other. What will the student do when encountering a Y or a Δ configuration?

The presenter had obviously been taught from generalization (the equations) to specific. This is easier because the generalization is more compact. The teacher may have developed the equations for the class, but R_1 and R_2 were probably used from the start; it takes less time than starting with examples using real numbers. In any case, the teacher probably did all the work himself, depriving the students of discovery.

The class had done so well in presenting the math puzzles; what had happened to their insight, creativity and love of discovery? Their skills in Step 1 and Step 2 seemed to have disappeared in their own discipline. When dealing with math puzzles they were in a free, open-ended mode, but when asked to deal with civil or electrical engineering they reverted to the mode in which they were taught. Beginning teachers need to make ideas their own, and it seems difficult for them to get beyond the approaches they were taught. But it's possible with example and encouragement.

ARGUMENTS AGAINST TEACHING DISCOVERY

Why were the students not taught in the discovery mode in their own discipline? There are many arguments used against the teaching of discovery, and they seem to win most of the time. The following is a partial list.

1. *Letting the students discover takes too much time.* If the teacher is not practiced at Steps 3 and 4, the student may take so long that the teacher finally just gives the solution. Then the whole exercise seems a waste of time. With the pressure to cover all the material, teaching discovery is usually the first casualty. But if the teacher *is* adept at Steps 3 and 4, a few concepts can be taught through discovery in a reasonable time. Certainly, there isn't time to present all concepts through discovery, but there must be some balance that doesn't exclude discovery entirely.

2. *The teacher is proud of his own clear views of things and wants to share them with the students.* It's a noble feeling to be turning night into day for the students, but much of the teacher's knowledge and insight doesn't get across to the students—the knowledge that came from going through the discovery experience. To a large extent we're kidding ourselves, and we find this out in the exams. It's an even better feeling to see the students actually sharing the discovery experience.

3. *Academia is not the place for disorderly methods. Students should be taught the tools, and creativity will come naturally when they are engineers.* Engineers could be more creative and sooner creative if academia treated discovery as a basic skill to be taught. It hasn't, and one of the results is that "academic" sometimes means "having no practical purpose or use."[15] Research is needed to quantify the improved performance of engineers taught in the discovery mode.

4. *Underclassmen don't have enough tools to discover until the capstone design in their senior year.* There's discovery at all levels—starting at infancy; it's not an all-or-nothing proposition. The teacher has far more tools than the students, and it's often difficult to put himself in their position. There will be gaps in the students' experience at any level, and the teacher will need to provide background knowledge as necessary. (Or a conversation with the teacher of a prerequisite course may be in order.)

5. *Students prefer to absorb ideas passively; they don't want to be put in a position where their ignorance is exposed.* This attitude is mostly a result of the contrast between a course taught through discovery and the lecture courses the students are used to. They can become impatient, wondering why the teacher doesn't just tell them the answer rather than play games. The first time I tried to incorporate discovery into a course, I got a lot of negative feedback. "Wolaver doesn't know what he's doing." "This is the poorest, most disorganized course I've taken." I retreated quickly to the lecture mode. But I've gotten better at the Four Steps now, and the students' resistance to something new is being offset by the fun of discovery.

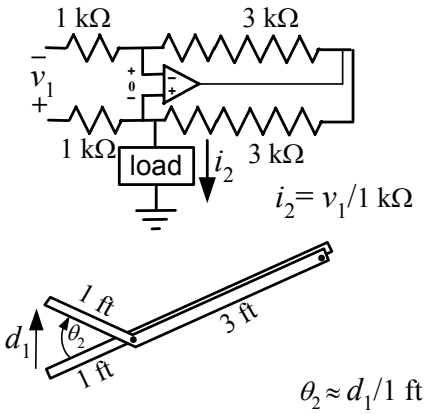
6. *It's hard to evaluate the students' discovery; there are too many paths they can take, and it's not clear which have merit.* If the student's result is correct, there shouldn't be a problem in evaluating it. But if a student misses the mark, it's hard to distinguish between confusion, a wrong approach, or perhaps a correct approach (which the teacher hadn't thought of) implemented incorrectly. Bruner observes, "It requires a sensitive teacher to distinguish an intuitive mistake—an interestingly wrong leap—from a stupid or ignorant mistake, and it requires a teacher who can give approval and correction simultaneously to the intuitive student."[16] This is where skill at Step 4 comes in. Occasionally, the student is using an approach outside the teacher's experience, and the teacher must become the student for the moment.

7. *Teaching discovery requires one-on-one interaction, and large classes don't allow that.* The ideal class size for interaction and brainstorming is around five. In larger classes, a panel of three to seven could be chosen to act as the discoverers while the rest observe. (The observers will jump into the action from time to time.) Discovery problems assigned as homework don't depend on the size of the class. But discovery still needs interaction with the teacher, and this help can be provided through email as students are doing the homework.

8. *Most results are already in the textbook, spoiling the surprise of discovery.* The order of the discovery process (numeric examples, to patterns, to guesses, to proof) is considerably different from the presentation in the book (assertion of an equation, then proof by symbols, then numeric examples). This difference often makes it possible to surprise the students even if they know the equation. Just as often, though, the textbook can take away the surprise. The teacher may want the students to guess the shape of a function—the probability density of the waiting time to the next event in a Poisson process—just from the fact that its shape later is the same as its shape earlier, differing only by a factor. If the students have already read that the probability density is a decaying exponential, they have a pretty easy time "guessing" that function. Fortunately, students usually don't read about a topic until after the class—when they have to start the homework.

9. *The discovery behind a concept or design may not be known—lost in history.* Sometimes the teacher will have to reconstruct or "reverse engineer" a design, guessing how the inventor came up with the solution. In his book reviewing mathematical discoveries, Polya says, "I cannot tell the true story how the discovery did happen, because nobody really knows that. Yet I shall try to make up a lively story how the discovery could have happened."[17] The circuit below delivers to the load a current proportional to the differential input voltage. How was this invented (discovered)? It looks similar to the differential amplifier pictured above, but the tips of the

"scissors" are welded together. When the "handles" are pulled apart, the scissors break creating an angle (a current). Perhaps the inventor worked with just such an analogy. I don't know.



Sometimes the path of discovery leading to a famous invention has been recorded.[18] A study of the history of science and engineering is useful in any case.

CONCLUSIONS

This started out as an exercise for myself to find the best I could do with teaching discovery and what the worst roadblocks were. I wanted to see whether the methods were sufficient to overcome the roadblocks. Can discovery and intuition be taught? The answer for me isn't a simple "yes" or "no," but rather "to some extent, and more than before." Every teacher will bring a different experience and perspective to the question and reach his or her own balance.

Step 1 of teaching discovery appeals to the teacher's own skills at learning—the love of playing around with a new concept or problem. He tries specific numbers, sees how the results depend on the numbers, looks for familiar patterns, guesses at a solution, and confirms it. With this in mind (Step 2), the teacher guides the student in a similar path (Step 3), watching for a similar sense in the student. The poor results can be frustrating if the teacher doesn't understand the student's sense of things (Step 4).

This description of teaching discovery is supposed to make it sound easy, but it isn't, of course. This paper provides no simple tricks that will insure a quick and deeply meaningful learning experience. (If such tricks existed, they would have been discovered and universally implemented long ago.) Instead, the teacher must discover the steps himself through trial and error. Habitual curiosity helps with Step 1. Thinking out loud to someone or writing down your thought process[14] helps with Step 2. Frustration with Step 3 is usually the result of the teacher expecting the students to have background knowledge, experiences, and concepts that they actually don't. This is uncovered in Step 4, which requires patience, careful listening, and imaginative guessing on the teacher's part.

The main barrier to teaching discovery is the amount of time it takes. Initially it will probably take four to six times as long to convey an idea or solution by letting the students discover it (compared with the expository lecture). But with practice, this can be reduced to perhaps a factor of two. This is a small price to pay for the students learning to think, not just learning equations. Obviously this approach can't be used for all the material; there's not enough time. Certain concepts can be selected for teaching by discovery, and others must be dropped to make room. The balance depends on the teacher's inclination, creativity, and experience.

So teaching through discovery will require more time. And this in the face of pressures to cover the material, catch up on grading, and get back to neglected research! With the prospect of losing tenure or a spouse, teachers head for the safe haven of traditional lecturing. So maybe this paper is missing the point; we should instead be researching the free-market forces. Do students taught through discovery perform so markedly better that demand for such students prompts school administrators to encourage this teaching method?

I believe that teachers will continue to seek better methods of teaching, with or without support; it's part of the passion of teaching. And discovery lies at the very heart of that passion.

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